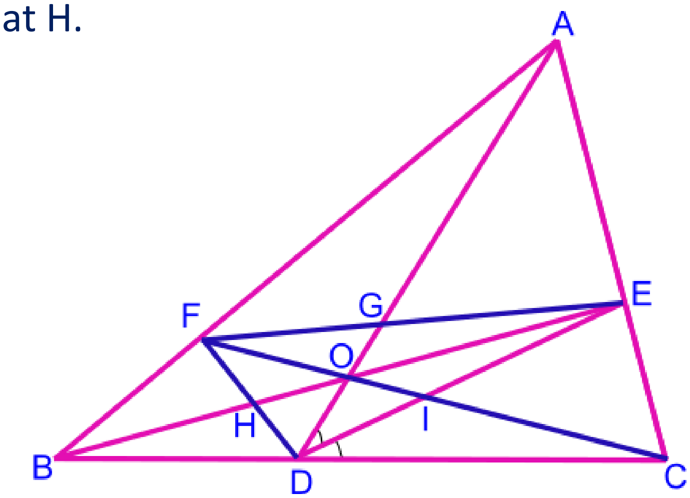
01.02.2024 - Cash Award Math Rider

Author's Solution

Construction :

Join and produce CO to meet AB at F. Let DE & CO cut at I. Join DF & EF. Let AO & EF cut at G and let BO & DF cut at H.



Proof:

In ΔODC, DI is the angle bisector

$\therefore \frac{OD}{DC} = \frac{OI}{IC}$ --------------------------------(1)

But, as per Concurrency Theorem

(available in this site),

$\frac{OI}{IC} = \frac{OF}{FC}$ ----------------------------- (2)

(1) & (2) $\rightarrow \frac{OD}{DC} = \frac{OF}{FC}$

$\Longrightarrow$ DF is the external angle bisector of $\Delta\ ODC$ for its angle at D.

$\Longrightarrow$DF or DH is the bisector of $\angle ODB$

$\Longrightarrow \frac{OD}{BD} = \frac{OH}{HB}$ ---------------------- (3)

Now, in ΔABC, AD, BE & CF are cevians concurrent at O.

$\therefore$ As per Concurrency Theorem (available in this site),

$\frac{OD}{DA} = \frac{OG}{GA}$ ---------------------------------(4)

(1) +(3)+(4) $\longrightarrow$

$$\frac{OD}{DC} + \frac{OD}{BD} + \frac{OD}{DA} = \frac{OI}{IC} + \frac{OH}{HB} + \frac{OG}{GA}$$ -------------------------- (5)

But as per Unity Pieces Theorem (available in this site),

$$\frac{OI}{IC} + \frac{OH}{HB} + \frac{OG}{GA} = 1$$ ----------------------------(6)

(5) & (6) $\longrightarrow$

$$\frac{OD}{AD} + \frac{OD}{BD} + \frac{OD}{CA} = 1$$

$$\Longrightarrow \left[\frac{1}{AD} + \frac{1}{BD} + \frac{1}{CD}\right] = \frac{1}{OD}$$ ---------------------------Proved

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